TRUE TRANSFORMATIONS OF SPACETIME LENGTHS AND APPARENT TRANSFORMATIONS OF SPATIAL AND TEMPORAL DISTANCES. I. THE THEORY

Tomislav Ivezić
Ruđer Bošković Institute, P.O.B. 180, 10002 Zagreb, Croatia
ivezic@rudjer.irb.hr

It is shown in this paper that the difference between the two forms of relativity - the "true transformation (TT) relativity" and - the "apparent transformation (AT) relativity" is essentially caused by the difference in the concept of *sameness* of a physical system, i.e., of a physical quantity, for different, relatively moving, observers. In the "TT relativity" the same quantity for different inertial frames of reference is covariantly defined four-dimensional (4D) tensor quantity, which transforms according to the Lorentz transformation as the TT. In the "AT relativity" parts of a 4D tensor quantity are often considered as the same quantity for different observers, although they correspond to different quantities in 4D spacetime, and they are not connected by the Lorentz transformation than by the AT. Then the true transformations of a spacetime length and the apparent transformations of a spatial distance (the Lorentz contraction) and of a temporal distance (the usual dilatation of time) are examined in detail. It is proved that only the true transformations of the spacetime length are in agreement with the special relativity as the theory of a 4D spacetime with the pseudo-Euclidean geometry.

PACS number(s): 03.30.+p

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality. H. Minkowski

A quantity is therefore physically meaningful (in the sense that it is of the same nature to all observers) if it has tensorial properties under Lorentz transformations. F. Rohrlich

I. INTRODUCTION

In [1] and [2] (see also [3]) two forms of relativity are discussed, the "true transformations (TT) relativity" and the "apparent transformations (AT) relativity." The notions of the TT and the AT are first introduced by Rohrlich [4], and, in the same meaning, but not under that name, discussed in [5] too.

The TT are the transformations of the four-dimensional (4D) spacetime tensors referring to the same quantity (in 4D spacetime) considered in different inertial frames of reference (IFRs), or in different coordinatizations of some IFR. An example of the TT are the covariant Lorentz transformations (LT) of 4D tensor quantities, (see [6] and [2]). Such covariant LT as the TT are the transformations in 4D spacetime and they transform some 4D tensor quantity $Q_{b..}^{a..}(x^c, x^d, ..)$ from an IFR S to $Q_{b..}^{\prime a..}(x^{\prime c}, x^{\prime d}, ..)$ in relatively moving IFR S', (all parts of the quantity are transformed). Since the "TT relativity" is based on the TT of

physical quantities it is obviously the manifestly covariant formulation of the special relativity. However, there is an important difference between the usual covariant formulation of the special relativity and the "TT relativity." Namely, it is considered in the usual covariant formulation of the special relativity that the 4D spacetime tensor quantities, e.g., the electromagnetic field tensor F^{ab}, do have well-defined mathematical meaning in the theory but, nevertheless, the real physical meaning in experiments is assumed to be attributed not to such 4D spacetime quantities than to some related quantities, e.g., the electric and magnetic three-vectors (3-vectors) $\mathbf{E}$ and $\mathbf{B}$, from the "3+1" space and time. In the "TT relativity" only one sort of quantities, the 4D spacetime tensor quantities, do have well-defined meaning in our 4D spacetime, both, mathematical meaning in the theory, and a real physical meaning in experiments; *the complete and well-defined measurement from the "TT relativity" viewpoint is such measurement in which all parts of some 4D quantity are measured.*

In contrast to the TT *the AT are not the transformations of 4D spacetime tensors and they do not refer to the same 4D quantity, but to different quantities in 4D spacetime. Usually, depending on the used AT, only a part of a 4D tensor quantity is transformed by the AT, and such a part of a 4D quantity, when considered in different IFRs (or in different coordinatizations of some IFR), corresponds to different quantities in 4D spacetime. Thus, for example, the AT refer to the quantities defined by the same way of measurement in different IFRs.* An example of the "AT relativity" is the conventional special relativity based on two Einstein's postulates and, consequently, on the relativity of simultaneity, on the synchronous definition of *the spatial length*, i.e., on the AT of the spatial length (the Lorentz contraction, see [4,5,1–3]), and the AT of *the temporal distance* (the conventional dilatation of time), as will be proved in this paper, and, as shown in [1] (see also [3]), on the AT of *the electric and magnetic 3-vectors* $\mathbf{E}$ and $\mathbf{B}$ (the conventional transformations of $\mathbf{E}$ and $\mathbf{B}$, see, e.g., [7], Sec.11.10).

In this paper, Sec.2, some general consideration on the two forms of relativity will be done. In Sec.3 the TT of the spacetime length for - a moving rod, Sec.3.1, and - a moving clock, Sec.3.2, are exposed. Then in Sec.4 the AT of - the spatial distance (the Lorentz "contraction"), Sec.4.1, and of - the temporal distance (the time "dilatation"), Sec.4.2, are considered. Conclusions are given in Sec.5.

II. GENERAL CONSIDERATION ON THE "TT RELATIVITY" AND THE "AT RELATIVITY"

The main difference between the "TT relativity" and the "AT relativity" stems from the difference in the concept of *sameness* of a physical system, i.e., of a physical quantity, for different observers. That concept actually determines the difference in what is to be understood as a relativistic theory.

In the "TT relativity" as Rohrlich [4] states: "The special theory of relativity is characterized by the group of Lorentz transformations which describes the way two different observers relate their observations of *the same physical systems*." (my emphasis) Then he continues with the words taken here as a second motto: "A quantity is therefore physically meaningful (in the sense that it is of the same nature to all observers) if it has tensorial properties under Lorentz transformations." Similarly Gamba states in [5]: "Special rela-

tivity gives us rules to compare results of an experiment performed by an observer S with results obtained by another observer S', moving with constant velocity with respect to S. It is, of course, implied that both obsevers are experimenting upon the *same* physical system ... ," and "The quantity $A_\mu(x_\lambda, X_\lambda)$ for S is the same as the quantity $A'_\mu(x'_\lambda, X'_\lambda)$ for S' when all the primed quantities are obtained from the corresponding unprimed quantities through Lorentz transformations (tensor calculus)."

These examples show that in the the "TT relativity" the special relativity is understood as the theory of 4D spacetime with pseudo-Euclidean geometry. Quantities of physical interest, both local and nonlocal, are represented by spacetime tensors, i.e., as covariant quantities, and the laws of physics are written in a manifestly covariant way as tensorial equations. Such an understanding of the special relativity and of the concept of *sameness* of a physical system, i.e., of a physical quantity, is consistently applied in [1], [2] (see also [3,8,9]) by extending the works [4] and [5] to the relativistic electrodynamics in terms of the introduction of the four-vectors (4-vectors) E^α and B^α of the electric and magnetic field, respectively, and their TT, instead of the usual 3-vectors $\mathbf{E}$ and $\mathbf{B}$ and their AT (the conventional transformations of $\mathbf{E}$ and $\mathbf{B}$, [7]). It has to be noted that although Rohrlich [4] and Gamba [5] clearly exposed the concept of sameness of a physical quantity in 4D spacetime they also did not notice that the usual transformations of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ [7] are - the AT, i.e., that $\mathbf{E}$ in S and $\mathbf{E}'$ in S' do not refer to the same 4D tensor quantity. The covariant formulation of electrodynamics with 4-vectors E^α and B^α is constructed in [1–3] and shown to be equivalent to the usual covariant formulation with the electromagnetic field tensor $F^{\alpha\beta}$. Also the covariant Majorana form of Maxwell's equations is done in [2], while in [1] (and [9]) the covariant form of the energy-momentum density tensor $T^{\alpha\beta}$ for the electromagnetic field and the fully covariant form of a nonlocal quantity, the electromagnetic momentum 4-vector P_f^α, are constructed in terms of the 4-vectors E^α and B^α. It has to be noticed once again that in the "TT relativity" only such 4D tensor quantities as are E^α and B^α, $F^{\alpha\beta}$, $T^{\alpha\beta}$, P_f^α, ..., are considered to be well-defined not only mathematically but also experimentally, as measurable quantities with real physical meaning.

The laws of physics written as tensorial equations with 4D spacetime tensors in an IFR will have the same form in some other IFR, i.e., in new coordinates, if new and old coordinates are connected by those coordinate transformations (the TT) that leave the interval ds, and thus the pseudo-Euclidean geometry of the spacetime, unchanged. (It is explicitly shown in [2] that the AT - the Lorentz contraction, as a coordinate transformation in 4D spacetime, changes the infinitesimal spacetime distance ds.) In fact, it is more correct to say that the laws of physics will have the same form for those coordinate transformations that leave the form, i.e., the functional dependence, of the metric tensor unchanged. Then ds will also be unchanged under such coordinate transformations, but, generally, the reverse does not hold. This means that *in the reference frames that are connected by such coordinate transformations all physical phenomena will proceed in the same way, (taking into account the corresponding initial and boundary conditions), and thus there is no physical difference between them, what is the content of the principle of relativity. The existence or nonexistence of the group of transformations that assure the form-invariance of the metric tensor, and thus also the form-invariance of the covariant equations (physical laws), is completely determined by the spacetime geometry. Hence, one concludes that in the "TT relativity" the principle of relativity, in contrast to the Einstein formulation of the special relativity [10], is not a*

fundamental principle, than it is a simple consequence of the spacetime geometry.

(The geometry of the spacetime is generally defined by the invariant infinitesimal spacetime distance ds of two neighboring points,

$$ds^2 = dx^a g_{ab} dx^b. \tag{1}$$

I adopt the following convention with regard to indices. Repeated indices imply summation. Latin indices $a, b, c, d, ...$ are to be read according to the abstract index notation, see [11], Sec.2.4.. They designate geometric objects and they run from 0 to 3. Thus $dx^{a,b}$ and g_{ab}, and of course ds (1), are defined independently of any coordinate system, e.g., g_{ab} is a second-rank covariant tensor (whose Riemann curvature tensor R^a_{bcd} is everywhere vanishing; the spacetime of special relativity is a flat spacetime, and this definition includes not only the IFRs but also the accelerated frames of reference). Greek indices run from 0 to 3, while latin indices $i, j, k, l, ...$ run from 1 to 3, and they both designate the components of some geometric object in some coordinate chart, e.g., $x^\mu(x^0, x^i)$ and $x'^\mu(x'^0, x'^i)$ are two coordinate representations of the position 4-vector x^a in two different inertial coordinate systems S and S', and $g_{\mu\nu}$ is the 4×4 matrix of components of g_{ab} in some coordinate chart. Let the coordinate transformations from S to S' be $x'^\mu = x'^\mu(x^\nu)$. Then the metric tensor $g_{\mu\nu}$ transforms according to the law $g'_{\mu\nu}(x') = (\partial x^\alpha/\partial x'^\mu)(\partial x^\beta/\partial x'^\nu)g_{\mu\nu}(x(x'))$, and if the coordinate transformations are such that they leave the form, i.e., the functional dependence, of the metric tensor unchanged, then ds will necessarily be an invariant quantity under such coordinate transformations.)

Since the "TT relativity" deals on the same footing with all possible coordinatizations of a chosen reference frame (inertial or accelerated), *the second Einstein postulate referred to the constancy of the coordinate velocity of light also does not hold in the "TT relativity."* Only in Einstein's coordinatization ("e" coordinatization; when Einstein's synchronization of distant clocks and cartesian space coordinates x^i are used in an IFR S) the coordinate, one-way, speed of light is isotropic and constant.

Thus *the basic elements of the "TT relativity," as a covariant formulation of the special relativity, and of the usual Einstein's formulation of the special relativity, are quite different. Einstein's formulation is based on two postulates: the principle of relativity and the constancy of the velocity of light. In the "TT relativity" the primary importance is attributed to the geometry of the spacetime; it is supposed that the geometry of our 4D spacetime is a pseudo-Euclidean geometry in which only 4D tensor quantities do have real physical meaning.* (The similar ideas about the primary importance of the geometry of the spacetime, not only in the general relativity but also in the special relativity, instead of Einstein's postulates [10], are expressed in several modern treatments, e.g., [12].)

Einstein [10], and many others, considered that general laws of physics must be covariant, but for Einstein, and for the majority of physicists, such covariance of general laws does not necessarily mean that the physical quantities of the theory have to be defined in a covariant manner, as covariant 4D tensor quantities. Thus, for example, Einstein [10] introduced into the special relativity several quantities that are not covariantly defined and whose transformations are the AT. The examples are: the synchronously defined spatial length with the AT - the Lorentz contraction, the temporal distance with the AT - the time dilatation, which will be discussed here. He also used the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ in the formulation of

the relativistic electrodynamics, and derived their transformations, which are not the LT of some well-defined quantity in 4D spacetime and they do not refer to the same 4D quantity, i.e., they are also the AT, as shown in [1] (and [3]). *In all cases in which two quantities connected by the AT are considered to refer to the same physical quantity in 4D spacetime we have*, what Gamba [5] calls, *the case of mistaken identity.* To better explain this issue I quote Gamba's words, [5]: "As far as relativity is concerned, quantities like A_μ and $\mathcal{A}'_\mu$ (they are from different IFRs S and S', and *they are connected by the AT*, my remark) are different quantities, not necessarily related to one another. To ask the relation between $\mathcal{A}'_\mu$ and A_μ, from the point of view of relativity, is like asking what is the relation between the measurement of the radius of the Earth made by an observer S and the measurement of the radius of Venus made by an observer S'. We can certainly take the ratio of the two measures; what is wrong is the tacit assumption that relativity has something to do with the problem just because the measurements were made by *two* observers." (At this point I remark once again that neither Gamba, despite of such clear understanding of the concept of sameness of a physical system in 4D spacetime, did not notice that the usual transformations of the 3-vectors **E** and **B** are, in fact, the AT, and in that respect he also dealt with the case of mistaken identity.)

III. TRUE TRANSFORMATIONS OF THE SPACETIME LENGTH

The whole above consideration is performed in order to emphasize the importance of the geometry of 4D spacetime in the formulation of special relativity, and to point out general differences between the "TT relativity" and the "AT relativity." In the following sections these differences will be illustrated considering some specific examples, the spacetime length with its TT and then the spatial and temporal distances with their AT.

A. The spacetime length - for a moving rod

For the sake of completness we repeat (and in some measure expand) the main results for the spacetime length, and for the AT of the spatial distance, that were already found in [2]. The invariant spacetime length (the Lorentz scalar) between two points (events) in 4D spacetime does have definite physical meaning in the "TT relativity" and it is defined as (in the abstract index notation)

$$l = (l^a g_{ab} l^b)^{1/2}, \tag{2}$$

where $l^a (l^b)$ is the distance 4-vector between two events A and B, $l^a = l^a_{AB} = x^a_B - x^a_A$, $x^a_{A,B}$ are the position 4-vectors and g_{ab} is the metric tensor. Using different coordinatizations of a given reference frame one can find different expressions, i.e., different representations of the spacetime length l, Eq.(2). We shall consider l in two relatively moving IFRs S and S' and in two coordinatizations "e" and "r" in these IFRs, where "e" stands for Einstein's coordinatization in which Einstein's synchronization of distant clocks and cartesian space coordinates x^i are used in an IFR, and where "r" stands for "radio" coordinatization of an IFR in which "everyday" or "radio" synchronization of distant clocks is used, see [2].

(Different synchronizations are determined by the parameter ε in the relation $t_2 = t_1 + \varepsilon(t_3 - t_1)$, where t_1 and t_3 are the times of departure and arrival, respectively, of the light signal, read by the clock at A, and t_2 is the time of reflection at B, read by the clock at B, that has to be synchronized with the clock at A. In Einstein's synchronization convention $\varepsilon = 1/2$ and the measured coordinate velocity of light is constant and isotropic. A nice example of a non-standard synchronization is "everyday" or "radio" clock synchronization [14] in which $\varepsilon = 0$ and there is an absolute simultaneity; see also [15]).

For further purposes we shall also need a covariant 4D expression for pure LT when written in geometrical terms, see [6] and [2],

$$L^a{}_b \equiv L^a{}_b(v) = g^a{}_b - \frac{2u^a v_b}{c^2} + \frac{(u^a + v^a)(u_b + v_b)}{c^2 - u \cdot v}, \tag{3}$$

where u^a is the proper velocity 4-vector of a frame S with respect to itself, $u^a = cn^a$, n^a is the unit 4-vector along the x^0 axis of the frame S, and v^a is the proper velocity 4-vector of S' relative to S, and $u \cdot v = u^a v_a$. The form of the covariant 4D Lorentz transformations (3) is independent of the chosen synchronization, i.e., coordinatization of reference frames. With the use of (3) the transformation of covariantly defined physical quantities reduces to the evaluation of invariant scalar products, e.g., when $L^a{}_b$ is applied to the position 4-vector x^a one finds (in the abstract index notation)

$$x'^a = g^a{}_b x^b + \frac{[n \cdot x - (2\gamma + 1) v \cdot x/c]\, n^a + (n \cdot x + v \cdot x/c) v^a/c}{1 + \gamma}, \tag{4}$$

where $\gamma = -n \cdot v/c$.

When Einstein's synchronization of distant clocks and cartesian space coordinates x^i are used in an IFR S then, e.g., the geometric object g_{ab} is represented by the 4×4 matrix of components of g_{ab} in that coordinate chart, i.e., it is the Minkowski metric tensor $g_{\mu\nu,e} = diag(-1, 1, 1, 1)$. Then $L^a{}_b$ is represented by $L^\mu{}_{\nu,e}$, the usual expression for pure LT, but with v_e^i (the proper velocity 4-vector v_e^μ is $v_e^\mu \equiv dx_e^\mu/d\tau = (\gamma_e c, \gamma_e v_e^i)$, $d\tau \equiv dt_e/\gamma_e$ is the scalar proper-time, and $\gamma_e \equiv (1 - v_e^2/c^2)^{1/2}$) replacing the components of the ordinary velocity 3-vector $\mathbf{V}$. (In the usual form the LT connect two coordinate representations (in the "e" coordinatization) $x_e^\mu, x_e'^\mu$ of a given event. $x_e^\mu, x_e'^\mu$ refer to two relatively moving IFRs (with the Minkowski metric tensor) S and S',

$$x_e'^\mu = L^\mu{}_{\nu,e} x_e^\nu,\ L^0{}_{0,e} = \gamma_e,\ L^0{}_{i,e} = L^i{}_{0,e} = -\gamma_e v_e^i/c,$$
$$L^i{}_{j,e} = \delta_j^i + (\gamma_e - 1) v_e^i v_{je}/v_e^2.$$

Since $g_{\mu\nu,e}$ is a diagonal tensor the space x_e^i and time t_e ($x_e^0 \equiv ct_e$) components of x_e^μ do have their usual meaning. Then the geometrical quantity ds^2 (1) can be written in terms of its representation ds_e^2, with the separated spatial and temporal parts, $ds^2 = ds_e^2 = (dx_e^i dx_{ie}) - (dx_e^0)^2$, and the same happens with the spacetime length l (2), $l^2 = l_e^2 = (l_e^i l_{ie}) - (l_e^0)^2$. Such separation remains valid in other inertial coordinate systems with the Minkowski metric tensor, and in S' one finds $l^2 = l_e'^2 = (l_e'^i l_{ie}') - (l_e'^0)^2$, where $l_e'^\mu$ in S' is connected with l_e^μ in S by the LT $L^\mu{}_{\nu,e}$.

Let us also consider the above relations in the "r" coordinatization. By the same construction as in [14] we can find the relations between the base vectors in "r" and "e" coordinatizations. (We consider, as in [14] and [2] (but now in 4D spacetime), that the spacetime

is endowed with base vectors, the temporal and the spatial base vectors. The bases $\{e_\mu\}$, with the base vectors $\{e_0, e_i\}$, and $\{r_\mu\}$, with the base vectors $\{r_0, r_i\}$, are associated with "e" and "r" coordinatizations, respectively, of a given IFR.) The connection between the bases $\{e_\mu\}$ and $\{r_\mu\}$ is

$$r_0 = e_0,\ r_i = e_0 + e_i. \tag{5}$$

Then the metric tensor g_{ab} becomes $g_{\mu\nu,r}$ with

$$g_{00,r} = g_{0i,r} = g_{i0,r} = g_{ij,r}(i \neq j) = -1, g_{ii,r} = 0. \tag{6}$$

The knowledge of $g_{\mu\nu,r}$ enable us to find the transformation matrix between "r" and "e" coordinatizations.

In [12] Logunov derived the expression for the transformation matrix connecting differentials of physical (how he named it) time and distance $dX^{\overline{\mu}}$ with the coordinate ones dx^μ, ($dX^{\overline{\mu}} = \lambda_\nu^{\overline{\mu}} dx^\nu$, in his notation, [12] Sec.22.), and by the same matrix $\lambda_\nu^{\overline{\mu}}$ he connected a physicaly measurable tensor with the coordinate one. Thus in the approach of [12] there are physical and coordinate quantities for the same coordinatization of the considered IFR. In my opinion both $dX^{\overline{\mu}}$ and dx^ν are equally well "physical" and measurable quantities, and we can interpret that $dX^{\overline{\mu}}$ corresponds to the Einstein coordinatization of a given IFR, while dx^ν corresponds to some arbitrary coordinatization of the same IFR. Hence, in my interpretation of Logunov results his matrix $\lambda_\nu^{\overline{\mu}}$ is, actually, the transformation matrix between some arbitrary coordinatization and the "e" coordinatization. It has to be noted that although in the Einstein coordinatization the space and time components of the position 4-vector do have their usual meaning, i.e., as in the prerelativistic physics, and in ds_e^2 the spatial and temporal parts are separated, it does not mean that the "e" coordinatization does have some advantage relative to other coordinatizations and that the quantities in the "e" base are more physical.

The elements of $\lambda_\nu^{\overline{\mu}}$ [12], which are different from zero, are $\lambda_0^{\overline{0}} = (-g_{00})^{1/2}$, $\lambda_i^{\overline{0}} = (-g_{0i})(-g_{00})^{-1/2}$, $\lambda_i^{\overline{i}} = \left[g_{ii} - (g_{0i})^2/g_{00}\right]^{1/2}$. We actually need the inverse transformation $(\lambda_\nu^{\overline{\mu}})^{-1}$ (it will be denoted as T_ν^μ to preserve the similarity with the notation from [2]). Then the elements (that are different from zero) of the matrix T_ν^μ, which transforms the "e" coordinatization to the coordinatization determined by the metric tensor $g_{\mu\nu}$, are

$$\begin{aligned} &T_0^0 = (-g_{00})^{-1/2}, \quad T_i^0 = (g_{0i})(-g_{00})^{-1}\left[g_{ii} - (g_{0i})^2/g_{00}\right]^{-1/2}, \\ &T_i^i = \left[g_{ii} - (g_{0i})^2/g_{00}\right]^{-1/2}. \end{aligned} \tag{7}$$

Hence T_ν^μ, which transforms the "e" coordinatization to the "r" coordinatization, is found to be $T_\mu^\mu = -T_i^0 = 1$, and all other elements of T_ν^μ are $= 0$. Using that T_ν^μ we find

$$x_r^\mu = T_\nu^\mu x_e^\nu, \quad x_r^0 = x_e^0 - x_e^1 - x_e^2 - x_e^3, \quad x_r^i = x_e^i. \tag{8}$$

The LT $L^\mu{}_{\nu,r}$ in the "r" base can be easily found from (3) and the known $g_{\mu\nu,r}$, and the elements that are different from zero are

$$\begin{aligned} &x_r'^\mu = L^\mu{}_{\nu,r} x_r^\nu, \quad L^0{}_{0,r} = K, \quad L^0{}_{2,r} = L^0{}_{3,r} = K - 1, \\ &L^1{}_{0,r} = L^1{}_{2,r} = L^1{}_{3,r} = (-\beta_r/K), L^1{}_{1,r} = 1/K, \quad L^2{}_{2,r} = L^3{}_{3,r} = 1, \end{aligned} \tag{9}$$

where $K = (1 + 2\beta_r)^{1/2}$, and $\beta_r = dx_r^1/dx_r^0$ is the velocity of the frame S' as measured by the frame S (it is assumed that S' is moving relative to S along the common $x_e^1, x_e'^1-$ axes), $\beta_r = \beta_e/(1-\beta_e)$ and it ranges as $-1/2 \prec \beta_r \prec \infty$. Since $g_{\mu\nu,r}$, in contrast to $g_{\mu\nu,e}$, is not a diagonal metric tensor then in ds_r^2 the spatial and temporal parts are not separated, and the same holds for the spacetime length l, see [2] for the results in 2D spacetime. Expressing dx_r^μ, or l_r^μ, in terms of dx_e^μ, or l_e^μ, one finds that $ds_r^2 = ds_e^2$, and also, $l_r^2 = l_e^2$, as it must be.

Next we consider the spacetime length in two relatively moving IFRs S and S' and in two coordinatizations "e" and "r" in these IFRs, i.e., we consider it with respect to $\{e_\mu\}, \left\{e'_\mu\right\}$ and $\{r_\mu\}, \left\{r'_\mu\right\}$ bases.

First we consider in short the same example as in [2], i.e., we consider a particular choice for the 4-vector l_{AB}^a (in the usual "3+1" picture it corresponds to an object, a rod, that is at rest in an IFR S and situated along the common $x_e^1, x_e'^1-$ axes). For simplicity we work in 2D spacetime and the situation is pictured in Fig.1.

The base vectors are constructed as in [14] and [2], and here we expose this construction once again for the sake of clearness of the whole exposition. The temporal base vector e_0 is the unit vector directed along the world line of the clock at the origin. The spatial base vector by definition connects *simultaneous* events, the event "clock at rest at the origin reads 0 time" with the event "clock at rest at unit distance from the origin reads 0 time," and thus it is synchronization-dependent. The spatial base vector e_1 connects two above mentioned simultaneous events when Einstein's synchronization ($\varepsilon = 1/2$) of distant clocks is used. The temporal base vector r_0 is the same as e_0. The spatial base vector r_1 connects two above mentioned simultaneous events when "radio" clock synchronization ($\varepsilon = 0$) of distant clocks is used. All the spatial base vectors $r_1, r_1', ..$ are parallel and directed along an (observer-independent) light line. Hence, two events that are everyday ("r") simultaneous in S are also "r" simultaneous for all other IFRs.

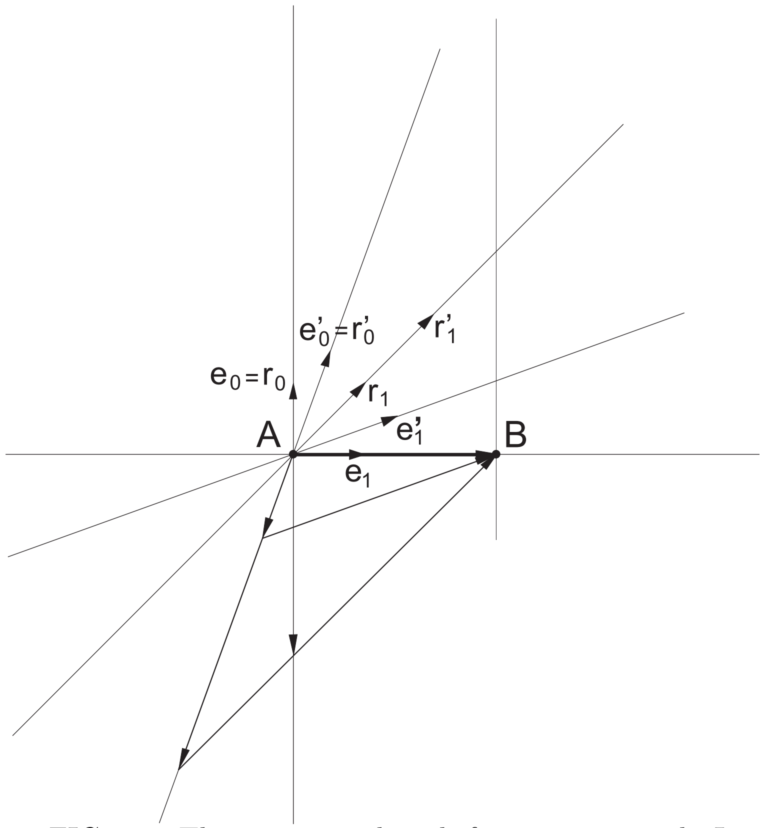

FIG. 1. The spacetime length for a moving rod. In the "TT relativity" the same quantity for different observers is the geometrical quantity, the distance 4-vector $l_{AB}^{a}=x_{B}^{a}-x_{A}^{a}$; only *one* quantity in 4D spacetime. It is decomposed with respect to $\{e_{\mu}\},\left\{e_{\mu}^{\prime}\right\}$ and $\{r_{\mu}\},\left\{r_{\mu}^{\prime}\right\}$ bases. The bases $\{e_{\mu}\},\left\{e_{\mu}^{\prime}\right\}$ refer to Einstein's coordinatization of two relatively moving IFRs S and S', and the bases $\{r_{\mu}\},\left\{r_{\mu}^{\prime}\right\}$ refer to the "radio" coordinatization of S and S'. l_{AB}^{a} corresponds, in the usual "3+1" picture, to an object, a rod, that is at rest in S and situated along the e_{1} base vector. The representation of l_{AB}^{a} in the $\{e_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\mu}=l_{e}^{0}e_{0}+l_{e}^{1}e_{1}=0e_{0}+l_{0}e_{1}$, in the $\left\{e_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\prime\mu}=-\beta_{e}\gamma_{e}l_{0}e_{0}^{\prime}+\gamma_{e}l_{0}e_{1}^{\prime}$, in the $\{r_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\mu}=-l_{0}r_{0}+l_{0}r_{1}$, and in the $\left\{r_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\prime\mu}=-Kl_{0}r_{0}^{\prime}+(1+\beta_{r})(1/K)l_{0}r_{1}^{\prime}$, where $K=(1+2\beta_{r})^{1/2}$, and $\beta_{r}=\beta_{e}/(1-\beta_{e})$.

The distance 4-vector $l_{AB}^{a}=x_{B}^{a}-x_{A}^{a}$ is decomposed with respect to $\{e_{\mu}\}$ base as

$$l_{AB}^{a}\rightarrow l_{AB,e}^{\mu}=l_{e}^{0}e_{0}+l_{e}^{1}e_{1}=0e_{0}+l_{0}e_{1}, \tag{10}$$

the temporal part of $l_{AB,e}^{\mu}$ is chosen to be zero. The spacetime length l is written in the $\{e_{\mu}\}$ base as $l=l_{e}=(l_{e}^{\mu}l_{\mu e})^{1/2}=(l_{e}^{i}l_{ie})^{1/2}=l_{0}$, as in the prerelativistic physics; it is in that case a measure of the spatial distance, i.e., of the rest spatial length of the rod. The observers in all other IFRs will look at the same events but associating with them different

coordinates and they all obtain the same value l for the spacetime length. The rest frame of the object, and the simultaneity of the events A and B in it, $l_e^0 = 0$, are chosen only to have the connection with the prerelativistic physics (and the "AT relativity"), which deals with "3+1" quantities and not with 4D quantities. In the "TT relativity," for the same rod at rest in S, we could take another choice for the 4-vector l_{AB}^a, e.g., the choice with $l_e^0 \neq 0$. The "TT relativity," unlike the nonrelativistic theory and the "AT relativity," is not interested in the spatial points, the front and the rear ends of the rod, but *in the events* A and B in the 4D spacetime. The decomposition of the chosen l_{AB}^a relative to the $\left\{e'_\mu\right\}$ base in S', (where in the "3+1" picture the rod is moving) is

$$l_{AB}^a \rightarrow l'^\mu_{AB,e} = -\beta_e \gamma_e l_0 e'_0 + \gamma_e l_0 e'_1. \tag{11}$$

Note that there is a dilatation of the spatial part $l'^1_e = \gamma_e l_0$ with respect to $l_e^1 = l_0$ and not the Lorentz contraction as predicted in the "AT relativity." Hovewer it is clear from the above discussion that comparison of only spatial parts of the two representations $l^\mu_{AB,e}$ and $l'^\mu_{AB,e}$ of the same physical quantity l_{AB}^a measured in two relatively moving IFRs S and S' respectively is physically meaningless in the "TT relativity." The invariant spacetime length of that object in S' is $l = l'_e = l_0$.

The distance 4-vector $l_{AB}^a = x_B^a - x_A^a$ is decomposed with respect to $\{r_\mu\}$ base as

$$l_{AB}^a \rightarrow l^\mu_{AB,r} = -l_0 r_0 + l_0 r_1, \tag{12}$$

and the TT length l is $l = l_r = l_0$, as it must be. In S' and in the $\left\{r'_\mu\right\}$ base l_{AB}^a is decomposed as

$$l_{AB}^a \rightarrow l'^\mu_{AB,r} = -K l_0 r'_0 + (1+\beta_r)(1/K) l_0 r'_1. \tag{13}$$

If only spatial parts of $l^\mu_{AB,r}$ and $l'^\mu_{AB,r}$ are compared than one finds that $\infty \succ l'^1_r \geq l_0$ for $-1/2 \prec \beta_r \leq 0$ and $l_0 \leq l'^1_r \prec \infty$ for $0 \leq \beta_r \prec \infty$, which once again shows that such comparison is physically meaningless in the "TT relativity." Hovewer the invariant spacetime length always takes the same value $l = l'_r = l_0$. Thus, as also seen from Fig.1, *one and the same geometrical quantity, the 4-vector* l_{AB}^a, *is considered in four different bases* $\{e_\mu\}, \left\{e'_\mu\right\}, \{r_\mu\}$ *and* $\left\{r'_\mu\right\}$, *where it is represented by its coordinate representations* $l_e^\mu, l'^\mu_e, l_r^\mu$ *and* l'^μ_r, *respectively.* An important conclusion emerges from the whole above consideration; *the usual 3D length of a moving object cannot be defined in the 4D spacetime of the TT relativity in an adequate way, since it is only the spatial length and not a 4D tensor quantity.*

B. The spacetime length - for a moving clock

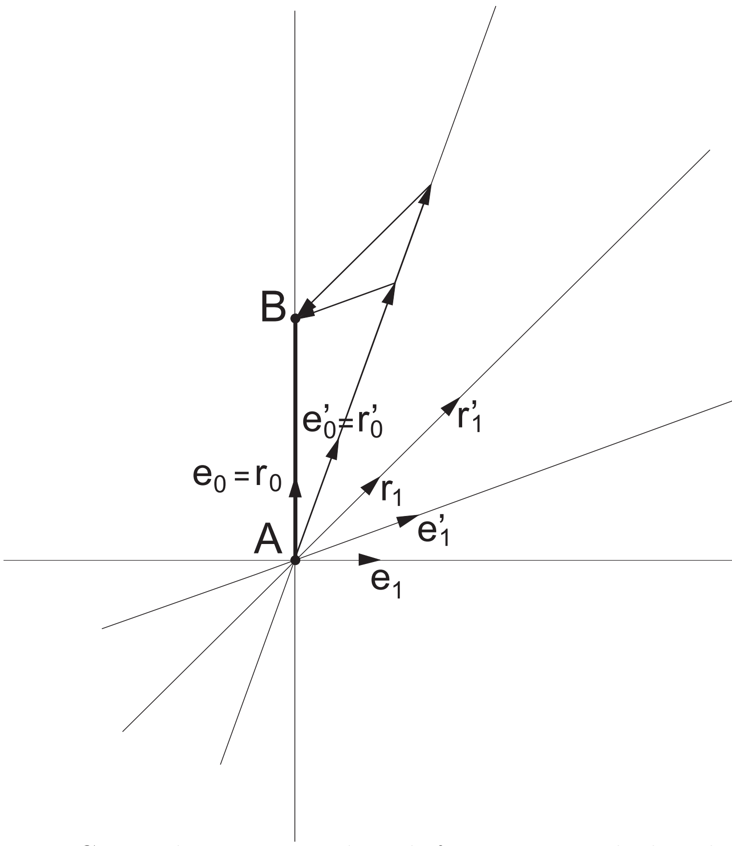

FIG. 2. The spacetime length for a moving clock. The same geometrical quantity, the distance 4-vector $l_{AB}^a = x_B^a - x_A^a$ is decomposed with respect to $\{e_\mu\}, \left\{e'_\mu\right\}$ and $\{r_\mu\}, \left\{r'_\mu\right\}$ bases. l_{AB}^a connects the events A and B (the event A represents the creation of the muon and the event B represents its decay after the lifetime τ_0 in S).and it is directed along the e_0 base vector from the event A toward the event B. The representation of l_{AB}^a in the $\{e_\mu\}$ base is $l_{AB}^a \rightarrow l_{AB,e}^\mu = c\tau_0 e_0 + 0e_1$, in the $\left\{e'_\mu\right\}$ base is $l_{AB}^a \rightarrow l_{AB,e}^{\prime\mu} = \gamma c\tau_0 e'_0 - \beta\gamma c\tau_0 e'_1$, in the $\{r_\mu\}$ base is $l_{AB}^a \rightarrow l_{AB,r}^\mu = c\tau_0 r_0 + 0r_1$, and in the $\left\{r'_\mu\right\}$ base is $l_{AB}^a \rightarrow l_{AB,r}^{\prime\mu} = Kc\tau_0 r'_0 - \beta K^{-1} c\tau_0 r'_1$.

Another example, i.e., another particular choice for the 4-vector l_{AB}^a, is presented in Fig.2. It clearly reveals the fundamental difference between the "TT relativity" and the "AT relativity." This example can be interpreted as the well known "muon experiment," but now considered in the "TT relativity." Again, as in the preceding section, the spacetime length and l_{AB}^a will be examined in two relatively moving IFRs S and S' and in two coordinatizations "e" and "r" in these IFRs, i.e., in $\left\{e_\mu\right\}, \left\{e'_\mu\right\}$ and $\{r_\mu\}, \left\{r'_\mu\right\}$ bases. The S frame is chosen to be the rest frame of the muon. Two events are considered; the event A represents the creation of the muon and the event B represents its decay after the lifetime τ_0 in S. The position 4-vectors of the events A and B in S, which are now taken to be on the world line of

a standard clock that is at rest in the origin of S, are decomposed with respect to $\{e_\mu\}$ base as $x_A^a \rightarrow x_{Ae}^\mu = x_{Ae}^0 e_0 + x_{Ae}^1 e_1 = 0e_0 + 0e_1$, and $x_B^a \rightarrow x_{Be}^\mu = x_{Be}^0 e_0 + x_{Be}^1 e_1 = c\tau_0 e_0 + 0e_1$. The distance 4-vector $l_{AB}^a = x_B^a - x_A^a$ that connects the events A and B is now directed along the e_0 base vector from the event A toward the event B. It is decomposed in the components in the $\{e_\mu\}$ base as

$$l_{AB}^a \rightarrow l_{AB,e}^\mu = l_e^0 e_0 + l_e^1 e_1 = c\tau_0 e_0 + 0e_1. \tag{14}$$

We see that in S and the $\{e_\mu\}$ base the position 4-vectors $x_{A,B}^a$ and the distance 4-vector l_{AB}^a have only temporal parts, i.e., $x_{Be}^1 = x_{Ae}^1 = l_e^1 = 0$, and the spacetime length l is $l = l_e = (-(l_e^0)^2)^{1/2} = (-c^2\tau_0^2)^{1/2}$; it is a measure of the temporal distance in S, as in the prerelativistic physics, and in this case one can speak about the lifetime τ_0 of the muon. In S', where this clock (muon) is moving, the position 4-vectors x_A^a and x_B^a of the events A and B, the creation and the decay of the muon respectively, are decomposed with respect to $\left\{e'_\mu\right\}$ base as $x_A^a \rightarrow x_{Ae}'^\mu = x_{Ae}'^0 e'_0 + x_{Ae}'^1 e'_1 = 0e'_0 + 0e'_1$, and $x_B^a \rightarrow x_{Be}'^\mu = x_{Be}'^0 e'_0 + x_{Be}'^1 e'_1 = \gamma c\tau_0 e'_0 - \beta\gamma c\tau_0 e'_1$, and the distance 4-vector l_{AB}^a is decomposed as

$$l_{AB}^a \rightarrow l_{AB,e}'^\mu = \gamma c\tau_0 e'_0 - \beta\gamma c\tau_0 e'_1. \tag{15}$$

Now in the $\left\{e'_\mu\right\}$ base l_{AB}^a contains not only the temporal part but also the spatial part and again the comparison of only the temporal parts of the distance 4-vector is physically meaningless in the "TT relativity," in contrast to the consideration in the "AT relativity." The notion of the time dilatation, which is in the "AT relativity" based on the comparison of only the temporal parts, is meaningless in the "TT relativity." However the correctly defined quantity is again the spacetime length, and in S' this length is $l = l'_e = l_e = (-c^2\tau_0^2)^{1/2}$.

In a similar way as above we find that in the "r" coordinatization the position 4-vectors of the events A and B in S are decomposed in the $\{r_\mu\}$ base as $x_A^a \rightarrow x_{Ar}^\mu = x_{Ar}^0 r_0 + x_{Ar}^1 r_1 = 0r_0 + 0r_1$, and $x_B^a \rightarrow x_{Br}^\mu = c\tau_0 r_0 + 0r_1$, and the distance 4-vector $l_{AB}^a = x_B^a - x_A^a$ is decomposed as

$$l_{AB}^a \rightarrow l_{AB,r}^\mu = l_r^0 r_0 + l_r^1 r_1 = c\tau_0 r_0 + 0r_1, \tag{16}$$

and the TT length l is $l = l_r = l_e$, as it must be. In S' and in the $\left\{r'_\mu\right\}$ base the position 4-vectors of the events A and B are $x_A^a \rightarrow x_{Ar}'^\mu = 0r'_0 + 0r'_1$ and $x_B^a \rightarrow x_{Br}'^\mu = x_{Br}'^0 r'_0 + x_{Br}'^1 r'_1 = (1+2\beta_r)^{1/2} c\tau_0 r'_0 - \beta_r(1+2\beta_r)^{-1/2} c\tau_0 r'_1$, and the components $l_r'^\mu$ of the distance 4-vector l_{AB}^a are equal to the components $x_{Br}'^\mu$, i.e., $l_{AB,r}'^\mu = x_{Br}'^\mu$. Thus

$$l_{AB}^a \rightarrow l_{AB,r}'^\mu = (1+2\beta_r)^{1/2} c\tau_0 r'_0 - \beta_r(1+2\beta_r)^{-1/2} c\tau_0 r'_1. \tag{17}$$

Comparing the temporal parts of $l_{AB,r}^\mu$ and $l_{AB,r}'^\mu$ one finds that $l_r'^0$ is larger than $l_r^0 = c\tau_0$ for $0 \prec \beta_r \prec \infty$, and it is smaller than l_r^0 for $-1/2 \prec \beta_r \prec 0$. Speaking in the language of the "AT relativity" one could say that for the positive β_r there is a "time dilatation" while for $-1/2 \prec \beta_r \prec 0$ there is a "time contraction." Since there is no physical reason for the preference of one coordinatization over the other we once again conclude that the comparison of only temporal (or spatial) parts of the distance 4-vectors is physically meaningless. This example nicely reveals the untenability of the notions from the "AT relativity" (the "Lorentz

contraction" or the "time dilatation") in the "TT relativity" as the theory of 4D spacetime with pseudo-Euclidean geometry. However, e.g., the spacetime length is always correctly defined quantity that takes the same value in both IFRs and in both coordinatizations, $l = l'_r = l_r = l'_e = l_e$; it can be compared in a physically meaningful sense in the "TT relativity."

We see that the geometrical quantities, the 4-vectors, $x^a_{A,B}$, l^a_{AB} have different representations depending on the chosen IFR and the chosen coordinatization of that IFR, e.g., $x^\mu_{e,r}, x'^\mu_{e,r}$. Although the Einstein coordinatization is preferred by physicists due to its simplicity and symmetry it is nothing more "physical" than others, e.g., the "r" coordinatization.

IV. THE AT OF SPATIAL AND TEMPORAL DISTANCES

In this section we consider the same two examples as above but now from the point of view of the conventional, i.e., Einstein's [10] interpretations of *the spatial length* of the moving rod and *the temporal distance* for the moving clock.

A. The AT of the spatial distance or the Lorentz "contraction"

The AT of the spatial distance is already considered in detail in [2] and therefore, here, we only quote the main results and the definitions, and also illustrate the whole consideration by Fig.3. The same example, a rod at rest in S, is pictured in Fig.1 when treated in the "TT relativity," and in Fig.3 when treated in the "AT relativity." It is mentioned in [2] that the synchronous definition of *the spatial length*, introduced by Einstein [10], defines length as *the spatial distance* between two spatial points on the (moving) object measured by simultaneity in the rest frame of the observer.

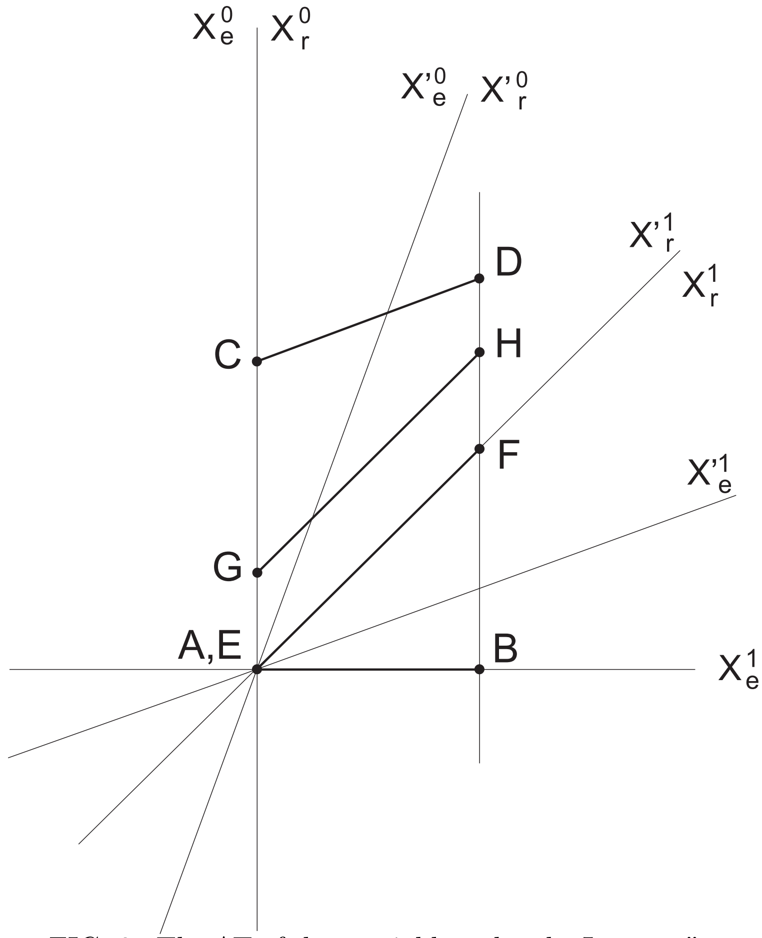

FIG. 3. The AT of the spatial length - the Lorentz "contraction" of the moving rod. The spatial distance $l^1_{ABe} = x^1_{Be} - x^1_{Ae} = l^1_e = l_0$ defines in the "AT relativity," and in the "e" base, the spatial length of the rod at rest in S, while $l'^1_{CDe} = x'^1_{De} - x'^1_{Ce} = l'^1_e$ is considered in the "AT relativity," and in the "e" base, to define the spatial length of the moving rod in S'. l'^1_e and $l^1_e = l_0$ are connected by the formulae for the Lorentz contraction of the moving rod $l'^1_e = l_0/\gamma_e$, with $t'_{Ce} = t'_{De} = t'_e = b$ and $t_{Be} = t_{Ae} = t_e = a$. a in S and b in S' are not related by the LT or any other coordinate transformation. Likewise in the "r" base, the spatial distance $l^1_{EFr} = x^1_{Fr} - x^1_{Er}$ defines in the "AT relativity" the spatial length of the rod at rest in S, while $l'^1_{GHr} = x'^1_{Hr} - x'^1_{Gr}$ defines the spatial length of the moving rod in S'. $l'^1_r = l'^1_{GHr}$ and $l^1_r = l^1_{EFr} = l_0$ are connected by the formulae for the Lorentz "contraction" of the moving rod in the "r" base $l'^1_r = l_0/K$ with $x'^0_{Hr} = x'^0_{Gr}$ and $x^0_{Fr} = x^0_{Er}$. In the "r" base there is a length dilatation $\infty \succ l'^1_r \succ l_0$ for $-1/2 \prec \beta_r \prec 0$ and the standard "length contraction" $l_0 \succ l'^1_r \succ 0$ for positive β_r, which clearly shows that the "Lorentz contraction" is not physically correctly defined transformation. In the "AT relativity" all four spatial lengths l^1_e, l'^1_e, l^1_r, l'^1_r are considered as the same quantity for different observers, but, in fact, they are four different quantities in 4D spacetime, and they are not connected by the Lorentz transformation.

Then, as explained in [2], *the spatial distance* $l^1_{ABe} = x^1_{Be} - x^1_{Ae}$ *defines in the "AT relativity," and in the "e" base, the spatial length of the rod at rest in* S, while $l'^1_{CDe} =$

$x'^1_{De} - x'^1_{Ce}$ *is considered in the "AT relativity," and in the "e" base, to define the spatial length of the moving rod in* S', see also Fig.3. From these definitions and considering only the transformation of the spatial part l^1_{ABe} of the distance 4-vector (in the "e" base) $l^\mu_{ABe} = x^\mu_{Be} - x^\mu_{Ae} = (0, l_0)$ one finds the relation between $l'^1_e = l'^1_{CDe}$ and $l^1_e = l^1_{ABe} = l_0$ (see [2]) as the famous formulae for the Lorentz contraction of the moving rod

$$l'^1_e = x'^1_{De} - x'^1_{Ce} = l_0/\gamma_e = (x^1_{Be} - x^1_{Ae})(1-\beta^2_e)^{1/2}, \quad t'_{Ce} = t'_{De}, t_{Be} = t_{Ae}, \tag{18}$$

where $\beta_e = v_e/c$, v_e is the relative velocity of S and S'. We see that the spatial lengths $l^1_e = l_0$ and $l'^1_e = l_0/\gamma_e$ refer not to the same 4D tensor quantity, as in the "TT relativity," but to two different quantities in 4D spacetime. These quantities are obtained by the same measurements in S and S'; the spatial ends of the rod are measured simultaneously at some $t_e = a$ in S and also at some $t'_e = b$ in S', and a in S and b in S' are not related by the LT or any other coordinate transformation.

The Lorentz "contraction" in the "r" coordinatization is also considered in [2] and pictured in Fig.3 here. The spatial ends of the considered rod, which is at rest in S, must be taken simultaneously in the "r" coordinatization too. Thus, in the "r" base, they must lie on the light line, i.e., on the x^1_r axis (that is along the spatial base vector r_1). The simultaneous events E and F (whose spatial parts correspond to the spatial ends of the rod) are the intersections of the x^1_r axis and the world lines of the spatial ends of the rod. The events E and F are not the same events as the events A and B, considered in the "e" base for the same rod at rest in S, since the simultaneity of the events is defined in different ways, see Fig.3. Therefore, in 4D spacetime the spatial length in the "r" base $l^1_r = l_0$ (with $x^0_{Fr} = x^0_{Er}$) is not the same 4D quantity as the spatial length in the "e" base $l^1_e = l_0$ (with $x^0_{Be} = x^0_{Ae}$). Applying the same procedure as in [2] one finds that *in the "r" base, the spatial distance* $l^1_{EFr} = x^1_{Fr} - x^1_{Er}$ *defines in the "AT relativity" the spatial length of the rod at rest in* S, *while* $l'^1_{GHr} = x'^1_{Hr} - x'^1_{Gr}$ *defines the spatial length of the moving rod in* S', see Fig.3. Using these definitions one finds the relation between $l'^1_r = l'^1_{GHr}$ and $l^1_r = l^1_{EFr} = l_0$ as the Lorentz "contraction" of the moving rod in the "r" base

$$l'^1_r = x'^1_{Hr} - x'^1_{Gr} = l_0/K = (1/K)(x^1_{Fr} - x^1_{Er}), \quad x'^0_{Hr} = x'^0_{Gr}, x^0_{Fr} = x^0_{Er}. \tag{19}$$

In contrast to the "e" coordinatization we find that in the "r" base there is a length dilatation $\infty \succ l'^1_r \succ l_0$ for $-1/2 \prec \beta_r \prec 0$ and the standard length "contraction" $l_0 \succ l'^1_r \succ 0$ for positive β_r, which clearly shows that the "Lorentz contraction" is not physically correctly defined transformation.

At the beginning of Sec.2 we have stated that the main difference between the "TT relativity" and the "AT relativity" stems from the difference in the concept of *sameness* of a physical quantity for different observers. This statement becomes clear when we compare Fig.1 and Fig. 3 and the considerations performed in Sec.3.1. and in this one. From Fig.1 and Sec.3.1. we see that *in the "TT relativity" the same quantity for different observers is the geometrical quantity, the 4-vector* l^a_{AB}; *only one quantity in 4D spacetime.* On the other hand from Fig.3 and the discussion in this section we see that in the "AT relativity" all four spatial lengths l^1_e, l'^1_e, l^1_r, l'^1_r are considered as the same quantity for different observers, but they are actually four different quantities in 4D spacetime.

Thus we conclude that - *the Lorentz contraction is the transformation that connects different quantities (in 4D spacetime) in different IFRs, which shows that it belongs to - the AT.*

B. The AT of the temporal distance or the time "dilatation"

In the "AT relativity" one can speak not only about the spatial distance but also about the temporal distance, and they are considered as well defined quantities. In Fig.4 we present the same "muon experiment" as in Fig.2.

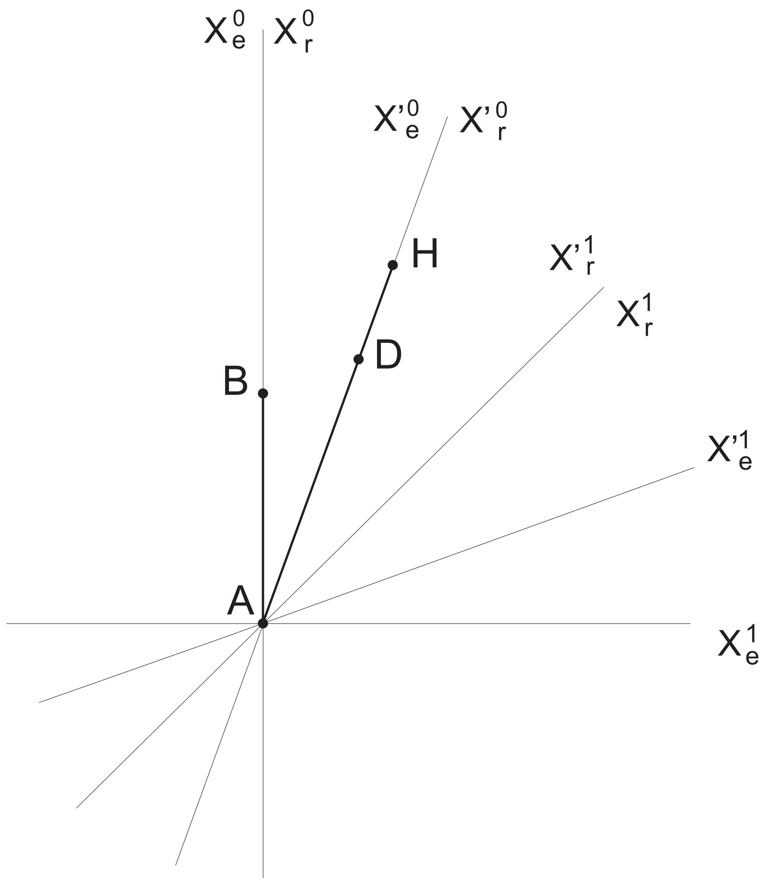

FIG. 4. The AT of the temporal distance - the "dilatation" of time for the moving clock. The temporal distance $l_{ABe}^{0}=l_{e}^{0}$ defines in the "AT relativity," and in the "e" base, the muon lifetime at rest, while $l_{ADe}^{\prime 0}=l_{e}^{\prime 0}$ is considered in the "AT relativity," and in the "e" base, to define the lifetime of the moving muon in $S^{\prime}$. The quantities $l_{e}^{\prime 0}$ and l_{e}^{0} are connected by the relation for the time dilatation, $l_{e}^{\prime 0}/c=t_{e}^{\prime}=\gamma_{e}l_{e}^{0}/c=\tau_{0}(1-\beta_{e}^{2})^{-1/2}$, with $x_{Be}^{1}=x_{Ae}^{1}$. Likewise, the temporal distance $l_{ABr}^{0}=l_{r}^{0}$ defines in the "AT relativity," and in the "r" base, the muon lifetime at rest, while $l_{AHr}^{\prime 0}=l_{r}^{\prime 0}$ is considered in the "AT relativity," and in the "r" base, to define the lifetime of the moving muon in $S^{\prime}$. $l_{r}^{\prime 0}$ and l_{r}^{0} are connected by the relation for the time "dilatation" in the "r" base $l_{r}^{\prime 0}=Kl_{r}^{0}=(1+2\beta_{r})^{1/2}c\tau_{0}$. The temporal separation $l_{r}^{\prime 0}$ in $S^{\prime}$, where the clock is moving, is smaller - "time contraction" - than the temporal separation $l_{r}^{0}=c\tau_{0}$ in S, where the clock is at rest, for $-1/2\prec\beta_{r}\prec 0$, and it is larger - "time dilatation" - for $0\prec\beta_{r}\prec\infty$. The "AT relativity" considers the temporal distances l_{e}^{0}, $l_{e}^{\prime 0}$, l_{r}^{0}, $l_{r}^{\prime 0}$ as the same quantity for different observers. However these temporal distances are really different quantities in 4D spacetime, and they are not connected by the Lorentz transformations.

Instead of to work with geometrical quantities $x_{A,B}^{a},l_{AB}^{a}$ and l one deals, in the "AT relativity," only with the spatial, or temporal, parts of their coordinate representations $x_{Ae,r}^{\mu},x_{Be,r}^{\mu}$ and $l_{e,r}^{\mu}$. First the "e" coordinatization, which is almost always used in the "AT relativity," is considered here. In 4D (at us 2D) spacetime and in the "e" coordinatization the events A and B are again on the world line of a muon that is at rest in S. The distance 4-vector (in the "e" base) is $l_{ABe}^{\mu}=x_{Be}^{\mu}-x_{Ae}^{\mu}=(c\tau_{0},0)$. Further one uses the Lorentz transformations to express $x_{Ae}^{\prime\mu}$, $x_{Be}^{\prime\mu}$, and $l_{ABe}^{\prime\mu}$ in $S^{\prime}$, in which the muon is moving, in terms of the corresponding quantities in S. This procedure yields $x_{A,Be}^{\prime 0}=ct_{A,Be}^{\prime}=\gamma_{e}(ct_{A,Be}-\beta_{e}x_{A,Be}^{1})$, and $x_{A,Be}^{\prime 1}=\gamma_{e}(\beta_{e}ct_{A,Be}-x_{A,Be}^{1})$, whence

$$
\begin{aligned}
l_{ABe}^{\prime 0}&=ct_{Be}^{\prime}-ct_{Ae}^{\prime}=\gamma_{e}(ct_{Be}-ct_{Ae})-\gamma_{e}\beta_{e}(x_{Be}^{1}-x_{Ae}^{1})\\
l_{ABe}^{\prime 1}&=x_{Be}^{\prime 1}-x_{Ae}^{\prime 1}=\gamma_{e}(x_{Be}^{1}-x_{Ae}^{1})-\gamma_{e}\beta_{e}(ct_{Be}-ct_{Ae}).
\end{aligned}
$$

Now comes the main difference between the two forms of relativity. Instead of to work with 4D tensor quantities and their LT (as in the "TT relativity") in the "AT relativity" the temporal part alone $l_{ABe}^{0}=c\tau_{0}$ of l_{ABe}^{μ} is considered as a well-defined quantity, and it defines the muon lifetime at rest. The existence of the spatial part l_{ABe}^{1} of l_{ABe}^{μ} is in this case completely neglected; one forgets the transformation of the spatial part l_{ABe}^{1} and considers only the transformation of the temporal part l_{ABe}^{0}. However, in the 4D (at us 2D) spacetime such an assumption means that in $S^{\prime}$ one actually does not consider the same events A and B as in S but some other two events C and D (in fact, in this specific example, the events A and D), see Fig.4. Then in the above transformation for l_{ABe}^{0} one has to write $x_{De}^{\prime 0}-x_{Ae}^{\prime 0}=l_{ADe}^{\prime 0}$ instead of $x_{Be}^{\prime 0}-x_{Ae}^{\prime 0}=l_{ABe}^{\prime 0}$. As we said the temporal distance $l_{ABe}^{0}=l_{e}^{0}$ defines in the "AT relativity," and in the "e" base, the muon lifetime at rest, while $l_{ADe}^{\prime 0}=l_{e}^{\prime 0}$ is considered in the "AT relativity," and in the "e" base, to define the lifetime of the moving muon in $S^{\prime}$. Taking that $x_{Be}^{1}=x_{Ae}^{1}$ in the equation for $l_{e}^{\prime 0}$ one finds the well-known relation for the time dilatation,

$$
l_{e}^{\prime 0}/c=t_{e}^{\prime}=\gamma_{e}l_{e}^{0}/c=\tau_{0}(1-\beta_{e}^{2})^{-1/2},\ \text{with } x_{Be}^{1}=x_{Ae}^{1}. \tag{20}
$$

As seen from Fig.4 the relation (20) connects two different quantities (in 4D spacetime) - the temporal parts of l_{ABe}^{μ} and $l_{ABe}^{\prime\mu}$, i.e., the temporal parts of the $\{e_{\mu}\}$ and $\left\{e_{\mu}^{\prime}\right\}$ representations of the distance 4-vector l_{AB}^{a}; the quantities l_{e}^{0} *and* $l_{e}^{\prime 0}$ *are different and, in fact, independent quantities in 4D spacetime, which are not connected by the Lorentz transformation.*

In the "r" coordinatization, see Fig.4., the $\{r_{\mu}\}$ representation of the distance 4-vector is $l_{ABr}^{\mu}=x_{Br}^{\mu}-x_{Ar}^{\mu}=(c\tau_{0},0)$. By means of the LT (3), when written in the $\{r_{\mu}\}$ base, we transform l_{ABr}^{μ} to $l_{ABr}^{\prime\mu}$ and find the relations

$$\begin{aligned}
l_{ABr}^{\prime 0}&=x_{Br}^{\prime 0}-x_{Ar}^{\prime 0}=K(x_{Br}^{0}-x_{Ar}^{0})=Kc\tau_{0},\\
l_{ABr}^{\prime 1}&=x_{Br}^{\prime 1}-x_{Ar}^{\prime 1}=(-\beta_{r}/K)(x_{Br}^{0}-x_{Ar}^{0})+(1/K)(x_{Br}^{1}-x_{Ar}^{1}).
\end{aligned}$$

Further, in the "r" base, one again forgets the transformation of the spatial part l_{ABr}^{1} of l_{ABr}^{μ}. In the same way as in the "e" base, in 4D (at us 2D) spacetime, such an assumption means that in $S^{\prime}$ one actually does not consider the same events A and B as in S but some other two events G and H, (in this particular example, the events A and H), see Fig.4. Then in the above transformation for $l_{ABr}^{0}=l_{r}^{0}$ one has to write $l_{AHr}^{\prime 0}$ instead of $l_{ABr}^{\prime 0}$; the temporal distance $l_{ABr}^{0}=l_{r}^{0}$ defines in the "AT relativity," and in the "r" base, the muon lifetime at rest, while $l_{AHr}^{\prime 0}=l_{r}^{\prime 0}$ is considered in the "AT relativity," and in the "r" base, to define the lifetime of the moving muon in $S^{\prime}$. The relation for the time "dilatation" in the "r" base becomes

$$l_{r}^{\prime 0}=Kl_{r}^{0}=(1+2\beta_{r})^{1/2}c\tau_{0}. \tag{21}$$

Again, as seen from Fig.4, the relation (21) connects two different quantities (in 4D spacetime) - the temporal parts of l_{ABr}^{μ} and $l_{ABr}^{\prime\mu}$, i.e., the temporal parts of the $\{r_{\mu}\}$ and $\left\{r_{\mu}^{\prime}\right\}$ representations of the distance 4-vector l_{AB}^{a}; the quantities l_{r}^{0} *and* $l_{r}^{\prime 0}$ *are different and, in fact, independent quantities in 4D spacetime, which are not connected by the Lorentz transformation.* We see from (21) that the temporal separation $l_{r}^{\prime 0}$ in $S^{\prime}$, where the clock is moving, is smaller - "time contraction" - than the temporal separation $l_{r}^{0}=c\tau_{0}$ in S, where the clock is at rest, for $-1/2\prec\beta_{r}\prec 0$, and it is larger - "time dilatation" - for $0\prec\beta_{r}\prec\infty$. In numerous papers and books the time dilatation given by (20) is considered as a fundamental relativistic effect, but as shown by the relation (21) the effect depends on the chosen coordinatizations, and as such cannot be the fundamental effect. In addition, we point out that from the point of view of the "TT relativity" such transformations that transform only some parts of 4D tensor quantities, and completely neglect the transformations of the remaining parts, are physically meaningless.

From Fig.2 and Sec.3.2. we see that *in the "TT relativity" the same quantity for different observers is the geometrical quantity, the 4-vector* l_{AB}^{a}; *only one quantity in 4D spacetime.* However from Fig.4 and this section we reveal that *in the "AT relativity" different quantities in 4D spacetime, the temporal distances* l_{e}^{0}, $l_{e}^{\prime 0}$, l_{r}^{0}, $l_{r}^{\prime 0}$, *are considered as the same quantity for different observers.*

One concludes from the above discussion that *both the Lorentz "contraction" and the time "dilatation" are the transformations connecting different quantities (in 4D spacetime) in different IFRs, and they both change the infinitesimal spacetime distance ds, i.e., the pseudo-Euclidean geometry of the 4D spacetime* (this is explicitly shown in [2] for the Lorentz

"contraction," and also it can be easily shown for the time "dilatation"). *Such characteristics of the Lorentz "contraction" and the time "dilatation" as the coordinate transformations show that both transformations belong to - the AT.*

V. CONCLUSIONS AND DISCUSSION

As shown in this paper the two forms of relativity the "TT relativity" and the "AT relativity" are substantially different theories. The concept of *sameness* of a physical quantity for different observers makes clear distinction between the two considered forms of relativity. In the "TT relativity" the same quantity for different observers is the geometrical quantity, only one quantity in 4D spacetime; in the case considered in this paper it is the 4-vector l_{AB}^a, (or the spacetime length l, the Lorentz scalar), as seen from Figs.1 and 2. In contrast to the "TT relativity" the traditionally used "AT relativity," considers different spatial lengths l_e^1, $l_e^{\prime 1}$, l_r^1, $l_r^{\prime 1}$, Fig.3 (the temporal distances l_e^0, $l_e^{\prime 0}$, l_r^0, $l_r^{\prime 0}$, Fig.4) as the same quantity for different observers. However, as seen from Fig.3 (Fig.4) the spatial lengths l_e^1, $l_e^{\prime 1}$, l_r^1, $l_r^{\prime 1}$ (the temporal distances l_e^0, $l_e^{\prime 0}$, l_r^0, $l_r^{\prime 0}$) are really different quantities in 4D spacetime, which are not connected by the Lorentz transformations $L^a{}_b$ (3), i.e., the quantities l_e^1 and $l_e^{\prime 1}$ (l_e^0 and $l_e^{\prime 0}$) are not connected by $L^\mu{}_{\nu,e}$, nor l_r^1 and $l_r^{\prime 1}$ (l_r^0 and $l_r^{\prime 0}$) are connected by $L^\mu{}_{\nu,r}$ ($L^\mu{}_{\nu,e}$ and $L^\mu{}_{\nu,r}$ are the representations of $L^a{}_b$ in the "e" and "r" coordinatizations, see Sec.3). Also, neither the quantities l_e^1 and l_r^1, etc., are connected by the coordinate transformation T_ν^μ, which transforms the "e" coordinatization to the "r" coordinatization, see Sec.3.

The consideration of the spacetime length (the "TT relativity") Sec.3, and of the spatial and temporal distances as well-defined quantities (the "AT relativity") Sec.4, reveals that only the "TT relativity" is in a complete agreement with the special relativity as the theory of 4D spacetime with pseudo-Euclidean geometry; when the 4D structure of our spacetime is correctly taken into account as in the "TT relativity" then there is no place either for the Lorentz "contraction" or for the time "dilatation," i.e., there is no place for the "AT relativity."

REFERENCES

[1] T.Ivezić, Found. Phys. Lett. **12**, 105 (1999).

[2] T.Ivezić, Found. Phys. Lett. **12**, (1999).

[3] T Ivezić, preprint SCAN-9802018 (on the CERN server).

[4] F. Rohrlich, Nuovo Cimento B **45**, 76 (1966).

[5] A. Gamba, Am. J. Phys. **35**, 83 (1967).

[6] D.E. Fahnline, Am. J. Phys. **50**, 818 (1982).

[7] J. D. Jackson, Classical Electrodynamics, 2nd ed. (Wiley, New York, 1977).

[8] T.Ivezić, preprint SCAN-9802017 (on the CERN server).

[9] T.Ivezić, preprint SCAN-9810078 (on the CERN server).

[10] A. Einstein, Ann. Physik **17,** 891 (1905), tr. by W. Perrett and G.B. Jeffery, in The principle of relativity (Dover, New York).

[11] R.M. Wald, General relativity (The University of Chicago Press, Chicago, 1984).

[12] A.A. Logunov, Lectures in the theory of relativity and gravity. A present-day analysis of the problem (Nauka, Moskva, 1987) (in Russian).

[13] F. Rohrlich, Phys. Rev. D **25**, 3251 (1982).

[14] C. Leubner, K. Aufinger and P. Krumm, Eur. J. Phys. **13,** 170 (1992).

[15] R. Anderson, I. Vetharaniam, G.E. Stedman, Phys. Rep. **295**, 93 (1998).

FIGURE CAPTIONS

Fig.1. The spacetime length for a moving rod. In the "TT relativity" the same quantity for different observers is the geometrical quantity, the distance 4-vector $l_{AB}^{a}=x_{B}^{a}-x_{A}^{a}$; only *one* quantity in 4D spacetime. It is decomposed with respect to $\{e_{\mu}\},\left\{e_{\mu}^{\prime}\right\}$ and $\{r_{\mu}\},\left\{r_{\mu}^{\prime}\right\}$ bases. The bases $\{e_{\mu}\},\left\{e_{\mu}^{\prime}\right\}$ refer to Einstein's coordinatization of two relatively moving IFRs S and S', and the bases $\{r_{\mu}\},\left\{r_{\mu}^{\prime}\right\}$ refer to the "radio" coordinatization of S and S'. l_{AB}^{a} corresponds, in the usual "3+1" picture, to an object, a rod, that is at rest in S and situated along the e_{1} base vector. The representation of l_{AB}^{a} in the $\{e_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\mu}=l_{e}^{0}e_{0}+l_{e}^{1}e_{1}=0e_{0}+l_{0}e_{1}$, in the $\left\{e_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\prime\mu}=-\beta_{e}\gamma_{e}l_{0}e_{0}^{\prime}+\gamma_{e}l_{0}e_{1}^{\prime}$, in the $\{r_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\mu}=-l_{0}r_{0}+l_{0}r_{1}$, and in the $\left\{r_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\prime\mu}=-Kl_{0}r_{0}^{\prime}+(1+\beta_{r})(1/K)l_{0}r_{1}^{\prime}$, where $K=(1+2\beta_{r})^{1/2}$, and $\beta_{r}=\beta_{e}/(1-\beta_{e})$.

Fig.2. The spacetime length for a moving clock. The same geometrical quantity, the distance 4-vector $l_{AB}^{a}=x_{B}^{a}-x_{A}^{a}$ is decomposed with respect to $\{e_{\mu}\},\left\{e_{\mu}^{\prime}\right\}$ and $\{r_{\mu}\},\left\{r_{\mu}^{\prime}\right\}$ bases. l_{AB}^{a} connects the events A and B (the event A represents the creation of the muon and the event B represents its decay after the lifetime τ_{0} in S).and it is directed along the e_{0} base vector from the event A toward the event B. The representation of l_{AB}^{a} in the $\{e_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\mu}=c\tau_{0}e_{0}+0e_{1}$, in the $\left\{e_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,e}^{\prime\mu}=\gamma c\tau_{0}e_{0}^{\prime}-\beta\gamma c\tau_{0}e_{1}^{\prime}$, in the $\{r_{\mu}\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\mu}=c\tau_{0}r_{0}+0r_{1}$, and in the $\left\{r_{\mu}^{\prime}\right\}$ base is $l_{AB}^{a}\rightarrow l_{AB,r}^{\prime\mu}=Kc\tau_{0}r_{0}^{\prime}-\beta K^{-1}c\tau_{0}r_{1}^{\prime}$.

Fig.3. The AT of the spatial length - the Lorentz "contraction" of the moving rod. The spatial distance $l_{ABe}^{1}=x_{Be}^{1}-x_{Ae}^{1}=l_{e}^{1}=l_{0}$ defines in the "AT relativity," and in the "e" base, the spatial length of the rod at rest in S, while $l_{CDe}^{\prime1}=x_{De}^{\prime1}-x_{Ce}^{\prime1}=l_{e}^{\prime1}$ is considered in the "AT relativity," and in the "e" base, to define the spatial length of the moving rod in S'. $l_{e}^{\prime1}$ and $l_{e}^{1}=l_{0}$ are connected by the formulae for the Lorentz contraction of the moving rod $l_{e}^{\prime1}=l_{0}/\gamma_{e}$, with $t_{Ce}^{\prime}=t_{De}^{\prime}=t_{e}^{\prime}=b$ and $t_{Be}=t_{Ae}=t_{e}=a$. a in S and b in S' are not related by the LT or any other coordinate transformation. Likewise in the "r" base, the spatial distance $l_{EFr}^{1}=x_{Fr}^{1}-x_{Er}^{1}$ defines in the "AT relativity" the spatial length of the rod at rest in S, while $l_{GHr}^{\prime1}=x_{Hr}^{\prime1}-x_{Gr}^{\prime1}$ defines the spatial length of the moving rod in S'. $l_{r}^{\prime1}=l_{GHr}^{\prime1}$ and $l_{r}^{1}=l_{EFr}^{1}=l_{0}$ are connected by the formulae for the Lorentz "contraction" of the moving rod in the "r" base $l_{r}^{\prime1}=l_{0}/K$ with $x_{Hr}^{\prime0}=x_{Gr}^{\prime0}$ and $x_{Fr}^{0}=x_{Er}^{0}$. In the "r" base there is a length dilatation $\infty\succ l_{r}^{\prime1}\succ l_{0}$ for $-1/2\prec\beta_{r}\prec0$ and the standard "length contraction" $l_{0}\succ l_{r}^{\prime1}\succ0$ for positive β_{r}, which clearly shows that the "Lorentz contraction" is not physically correctly defined transformation. In the "AT relativity" all four spatial lengths l_{e}^{1}, $l_{e}^{\prime1}$, l_{r}^{1}, $l_{r}^{\prime1}$ are considered as the same quantity for different observers, but, in fact, they are four different quantities in 4D spacetime, and they are not connected by the Lorentz transformation.

Fig.4. The AT of the temporal distance - the "dilatation" of time for the moving clock. The temporal distance $l_{ABe}^{0}=l_{e}^{0}$ defines in the "AT relativity," and in the "e" base, the muon lifetime at rest, while $l_{ADe}^{\prime0}=l_{e}^{\prime0}$ is considered in the "AT relativity," and in the "e" base, to define the lifetime of the moving muon in S'. The quantities $l_{e}^{\prime0}$ and l_{e}^{0} are connected by the relation for the time dilatation, $l_{e}^{\prime0}/c=t_{e}^{\prime}=\gamma_{e}l_{e}^{0}/c=\tau_{0}(1-\beta_{e}^{2})^{-1/2}$, with $x_{Be}^{1}=x_{Ae}^{1}$. Likewise, the temporal distance $l_{ABr}^{0}=l_{r}^{0}$ defines in the "AT relativity," and in the "r" base,

the muon lifetime at rest, while $l_{AHr}'^{0}=l_r'^{0}$ is considered in the "AT relativity," and in the "r" base, to define the lifetime of the moving muon in S'. $l_r'^{0}$ and l_r^0 are connected by the relation for the time "dilatation" in the "r" base $l_r'^{0}=Kl_r^0=(1+2\beta_r)^{1/2}c\tau_0$. The temporal separation $l_r'^{0}$ in S', where the clock is moving, is smaller - "time contraction" - than the temporal separation $l_r^0=c\tau_0$ in S, where the clock is at rest, for $-1/2\prec\beta_r\prec 0$, and it is larger - "time dilatation" - for $0\prec\beta_r\prec\infty$. The "AT relativity" considers the temporal distances l_e^0, $l_e'^{0}$, l_r^0, $l_r'^{0}$ as the same quantity for different observers. However these temporal distances are really different quantities in 4D spacetime, and they are not connected by the Lorentz transformations.